Übungsblatt 1

n+1 = O(n)

$n^2+n = O(n^2)$

$n + \sqrt{n} = O(n)$

logn + 20 = O(logn)

$$\sum_{i=0}^{n} i = 0 + 1 + \cdots + n = (0 + n) + \big(1 + (n - 1)\big) + \ \ldots = \frac{n+1}{2} * n = \frac{n^2 + n}{2} = O(n^2)$$

Aufgabe 1.1

$$\sum_{i=0}^{n} x^i = O(n)$$

1) x=1

   $$\sum_{i=0}^{n} 1^i = (n + 1) * 1 = O(n)$$

2) x>1

   $$\sum_{i=0}^{n} x^i > x^n$$

$x^n \notin O(n)$

3) 0<x<1

   Beweis durch Partialsummen

   1. $s_n = \sum x^i = x^0 + x^1 + x^2 + \cdots + x^n$

   2. $s_n = x + x^2 + x^3 + \cdots + x^{n+1}$

   1-2. $s_n - (x * s_n) = 1 - x^{n+1}$

   $s_n(1 - x) = 1 - x^{n+1}$

   $s_n = \frac{1 - x^{n+1}}{1 - x} \rightarrow \frac{1}{1 - x} (n \rightarrow \infty)$

   z.B. $\frac{1}{1-\frac{1}{2}} = 2 = O(1) = O(n)$

   ➔ Gleichung ist erfüllt für alle x<=1

Aufgabe 1.2.a

O(logP(n)) = O(logn)

P(n) = n^k

=> O(logn^k) = O(k*logn) = O(logn)

Aufgabe 1.2.b

$$\sum_{i=0}^{n} i^k < \sum_{i=0}^{n} n^k = (n+1) * n^k = n^{k+1} + n^k = O(n^{k+1})$$

Aufgabe 1.2.c

$n^{1,5}$ + n$\log_2$n = O($n^{1,5}$)

$n\sqrt{n} + nlog_2 n = n(\sqrt{n} + log_2 n) \leq n(2\sqrt{n})$ für $n \to \infty$

$$\frac{d}{dn} log_2 n = \frac{d}{dn}\frac{lnn}{ln2} = \frac{d}{dn}\left(\frac{1}{ln2} * lnn\right) = \frac{1}{ln2}\frac{1}{n} = \frac{1}{n * ln2} < \frac{1}{2\sqrt{n}} = \frac{d}{dn}\sqrt{n}$$

Aufgabe 1.2.d

$$\left.\begin{array}{l} f(n) = O(s(n)) \\ \\ g(n) = O(r(n)) \end{array}\right\} \quad => f(n)-g(n) = O(s(n)-r(n))$$

Gegenbeispiel:

f(n) = 4n+1   s(n) = n+1   O(s(n)) = O(n)   O(f(n)) = O(n)

g(n) = 3n   O(g(n)) = O(r(n)) = O(n)

$$\left.\begin{array}{l} f(n)-g(n) = n+1 = O(n) \\ \\ s(n)-r(n) = n+1-n = 1 = O(1) \end{array}\right\} \quad O(n) < O(1) \quad ↯$$

Aufgabe 1.2.e

$2^{(2^n)} = O(n^{(2^n)})$

n = 1

$2^{(2^1)} > 1^{(2^1)}$

n = 2

$2^{(2^2)} = 2^{(2^2)}$